Parameter Optimization of Frequency Estimation by Distribution Using Particle Swarm Optimization

Yuta OGATA, Ryo KABUTAN, and Takeshi NISHIDA

***Abstract*— Frequency estimation is essential for analyzing and controlling mechanical vibrations. Presently, the most popular method for dominant frequency estimation is the FFT (fast Fourier transform). However, this method requires several cycles of a signal and a longer time when estimating with low-frequency signals. Recently, a novel method, which uses a distribution technique called FED (frequency estimation by distribution) was proposed [1]. It is faster and offers a higher precision than the conventional methods. The method estimates the target frequency by using 21 to 30 sampled values without considering the sampling frequency. To use the FED, it is necessary to set some parameters in advance; however, the guidelines for the configuration of parameters are not clear. Thus, we focused on an optimization method for the FED parameters by using a meta-heuristic technique called particle swarm optimization. Furthermore, we demonstrated its applicability through several simulations.**

I. Introduction

Frequency estimation is essential for analyzing and controlling mechanical vibrations. It is an important technology, applicable to numerous fields, for example, aerospace, medical, construction, and machinery. Currently, the most popular method of dominant frequency estimation is the FFT (fast Fourier transform). However, this method requires several cycles of a signal and a longer time when estimating with low-frequency signals. Moreover, signal adjustments are usually required to be made. Recently, a novel method, which uses a distribution technique called FED (frequency estimation by distribution) has been proposed; it is faster and offers a higher precision than the conventional methods. The method estimates the target frequency by using 21 to 30 sampled values without considering the sampling frequency. To use the FED, it is necessary to set some parameters in advance; however, the guidelines for the configuration are not clear. Thus, we focused on an optimization method for the FED parameters by using a meta-heuristic method called PSO (particle swarm optimization). Furthermore, we demonstrated its applicability through several simulations.

II. Frequency Estimation by Distribution

A. Introducing the Gaussian function

Here, an outline of the FED method is described. We considered a periodic signal as follows:

$$x(t) = X_0 \cos(\omega t - \theta) \tag{1}$$

where X_0 is the amplitude, ω is the frequency, and θ is the delayed phase. The frequency can be calculated:

$$\ddot{x}(t) + \omega^2 x(t) = 0 \tag{2}$$

However, it is difficult to calculate the value of $\ddot{x}(t)$ with high accuracy because discrete sensor signals are processed in actual problems. Thus, we introduce a parameter estimation method, which uses a distribution function. An estimation target function and a distribution function are represented by $f(t)$ and $\phi(t)$, respectively. The distribution function is a C^∞ class function and has a compact support. The function satisfies following equation.

$$\lim_{t\to\pm\infty} \frac{\mathrm{d}^n \phi(t)}{\mathrm{d}t^n} = 0 \ (n \in \mathbb{Z}_{\geq 0}) \tag{4}$$

From the above equation, we obtain

$$\begin{aligned}\int_{-\infty}^{\infty} \frac{\mathrm{d}f(t)}{\mathrm{d}t}\phi(t)\mathrm{d}t &= [f(t)\phi(t)]_{-\infty}^{\infty} - \int_{-\infty}^{\infty} f(t)\frac{\mathrm{d}\phi(t)}{\mathrm{d}t}\mathrm{d}t \\ &= -\int_{-\infty}^{\infty} f(t)\frac{\mathrm{d}\phi(t)}{\mathrm{d}t}\mathrm{d}t\end{aligned}$$

Moreover, the following equation is deduced from the above equations.

$$\int_{-\infty}^{\infty} \frac{\mathrm{d}^n f(t)}{\mathrm{d}t^n}\phi(t)\mathrm{d}t = (-1)^n \int_{-\infty}^{\infty} f(t)\frac{\mathrm{d}^n \phi(t)}{\mathrm{d}t^n}\mathrm{d}t \tag{5}$$

This equation implies that the derivative of $f(t)$ can be represented using the derivative of $\phi(t)$.

Here, we consider the differential equation Eqn. (1). If we multiply both members by $\phi(t)$ and integrate them, the following equation is obtained.

$$\int_{-\infty}^{\infty} \ddot{x}(t)\phi(t)\mathrm{d}t + \omega^2 \int_{-\infty}^{\infty} x(t)\phi(t)\mathrm{d}t = 0 \tag{6}$$

Then, the following equation is obtained upon referring to Eqn. (5).

$$\int_{-\infty}^{\infty} x(t)\ddot{\phi}(t)\mathrm{d}t + \omega^2 \int_{-\infty}^{\infty} x(t)\phi(t)\mathrm{d}t = 0 \tag{7}$$

This equation may be transformed to give

$$\omega^2 = -\frac{\int_{-\infty}^{\infty} x(t)\ddot{\phi}(t)\mathrm{d}t}{\int_{-\infty}^{\infty} x(t)\phi(t)\mathrm{d}t} \tag{8}$$

Yuta OGATA, Ryo KABUTAN, Takeshi NISHIDA are with Kyushu Institute of Technology, Kitakyushu, Fukuoka, Japan (corresponding author's e-mail:nishida@cntl.kyutech.ac.jp)

The support of a Gaussian function is compact; hence, we can use it as a distribution function. Such a Gaussian function is represented as follows:

$$\phi(t) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(t-\mu)^2}{\sqrt{2\sigma^2}}\right\} \tag{9}$$

where μ is the mean value and σ^2 is the variance. The derivatives of this function are represented as follows:

$$\dot{\phi}(t) = -\frac{t-\mu}{\sigma^2}\phi(t) \tag{10}$$

$$\ddot{\phi}(t) = \frac{(t-\mu)^2 - \sigma^2}{\sigma^4}\phi(t) \tag{11}$$

Thus, the value of σ^2 in Eqn. (8) can be calculated using Eqns. (9) and (11).

B. Discretization

We have to consider measurement sampling in actual applications. Here, let N be the number of samplings, T_s be the sampling interval, $k = NT_s$ be the discrete time, and T_0 be the starting time for any round of measurements. We set the mean value of a Gaussian function to be used as a distribution function $\mu = NT_s/2$, and we set the value of σ^2 with a following limitation by a small number $\epsilon \in \mathbb{R}_{\geq 0}$

$$0 < |\phi(T_0)| = |\phi(T_0 + NT_s)| \ll \epsilon \tag{12}$$

The following approximation was found suitable for actual applications.

$$\int_{-\infty}^{\infty} x(t)\phi(t)\mathrm{d}t \cong \int_{T_0}^{T_0+2\mu} x(t)\phi(t)\mathrm{d}t \tag{13}$$

Next, we considered the calculation of Eqn. (8) in a discrete time window $[T_0, T_0 + NT_s]$ because the signal $x(t)$ is measured as a discrete signal $x(kT_s)$. Then, Eqn. (13) can be approximated by the sectional measurement method as follows:

$$\int_{T_0}^{T_0+2\mu} x(t)\phi(t)\mathrm{d}t \cong \sum_{k=0}^{N} x(T_0 + kT_s)\phi(T_0 + NT_s)T_s$$

From the relations mentioned above, Eqn. (8) can be represented as follows:

$$\omega^2 \cong -\frac{\sum_{k=0}^{N} x(T_0 + kT_s)\ddot{\phi}(T_0 + NT_s)}{\sum_{k=0}^{N} x(T_0 + kT_s)\phi(T_0 + NT_s)} \tag{14}$$

If the ideal calculation error is not taken into consideration, the estimation result does not change with respect to the value of θ in Eqn. (1). Thus, the value $T_0 \geq 0$ can be arbitrarily set, and the following equations hold

$$\begin{aligned}\omega^2 &\cong -\frac{\sum_{n=0}^{N} x(T_0 + nT_s)\ddot{\phi}(T_0 + nT_s)}{\sum_{n=0}^{N} x(T_0 + nT_s)\phi(T_0 + nT_s)} \\ &\cong -\frac{\sum_{n=0}^{N} x(T_0 + nT_s)\ddot{\phi}(nT_s)}{\sum_{n=0}^{N} x(T_0 + nT_s)\phi(nT_s)} \\ &\cong -\frac{\sum_{n=0}^{N} x(nT_s)\ddot{\phi}(nT_s)}{\sum_{n=0}^{N} x(nT_s)\phi(nT_s)}\end{aligned} \tag{15}$$

The values $\phi(t)$ and $\ddot{\phi}(t)$ do not need to be time updated; their initial values at time $[0, NT_s]$ can be used at all times.

$$\begin{aligned}\omega^2 &\cong -\frac{\sum_{n=0}^{N} x(T_0 + nT_s)\exp\left\{-\frac{(nT_s-\mu)^2}{2\sigma^2}\right\}\frac{(nT_s-\mu)^2-\sigma^2}{\sigma^4}}{\sum_{n=0}^{N}\sum_{n=0}^{N} x(T_0 + nT_s)\exp\left\{-\frac{(nT_s-\mu)^2}{2\sigma^2}\right\}} \\ &\cong -\frac{\sum_{n=0}^{N} x_n p_n q_n}{\sum_{n=0}^{N} x_n p_n}\end{aligned} \tag{16}$$

Moreover, by reduction of the constant component, the above equation could be expressed as follows:

$$\begin{aligned} x_n &\triangleq x(T_0 + nT_s) \\ p_n &\triangleq \exp\left\{-\frac{(nT_s-\mu)^2}{2\sigma^2}\right\} \\ q_n &\triangleq \frac{(nT_s-\mu)^2-\sigma^2}{\sigma^4}\end{aligned}$$

If the target frequency changes, the vector $\boldsymbol{x} \triangleq [x_0 \dots x_n \dots x_N]^T$ is updated by shifting T_0 according to the time evolution. Here, the estimation of ω cannot be executed until the matrix $\boldsymbol{x}$ is completely filled, i.e., the estimation is executed after time NT_s. In actual cases, the value of T_s depends on the measuring sensors and cannot change. Therefore, in this method, the values of N and σ have to be selected.

III. Parameter Optimization by OPSO

A. Target Parameters

The optimal value of the abovementioned parameter σ changes according to the frequency of the target signal. Moreover, a reduction in the number of data points N is required for fast calculation. The changing values of these parameters interfere with each other, their optimum values change in accordance with the sampling period, and hence, it is difficult to obtain the optimum values of these parameters analytically. Therefore, in this research, we optimized the parameters online by using a meta-heuristic method called the OPSO (Online PSO) algorithm. This method is applicable by considering time variation of the parameters. The m-th particle has two parameters: the first one is the size of the time window for the data and the second is the distribution:

$$\boldsymbol{z}_k^{(m)} \triangleq \left[N_k^{(m)}, \sigma_k^{(m)}\right]^T \tag{17}$$

Moreover, the estimation evaluation parameter is defined as follows:

$$e_k^{(m)} = \min \sum_{n=k-N_k^{(m)}}^{k} \left\{ \cos\left(\omega_k^{(m)} n T_s - \theta\right) - x(nT_s) \right\}^2 \quad (18)$$

For the procedure outlined in steps 1 and 3 (described below), we use a cost function defined as follows:

$$f_k\left(\boldsymbol{z}_k^{(m)}\right) = (1-\alpha) e_k^{(m)} - \alpha N_k^{(m)} \quad (19)$$

where $\alpha \in \mathbb{R}$ and $0 \leq \alpha \leq 1$. Moreover, $\omega_k^{(m)}$ is the estimated angular frequency that is based on $\boldsymbol{z}_k^{(m)}$. The cost function implies that by using the parameters of the particle $\boldsymbol{z}_i$, the value of the cost function decreases when the estimation error is small and the number of datapoints is smaller.

B. OPSO

The PSO [2] is a meta-heuristic method in which solutions are obtained by moving numerous search points, called particles, over the search space based on the past action history and the dynamically adjusted velocity. Consider the optimization of the following function $f: \mathbb{R}^l \to \mathbb{R}$.

$$\min_{\boldsymbol{z}} f(\boldsymbol{z}) \geq 0 \quad (20)$$

We adopt the OPSO algorithm [3] wherein a time-varying evaluation function $f_k(\cdot)$ is used to explicitly express environmental changes. The position and velocity of particles in the search space are denoted by $\boldsymbol{z}_k^{(m)} \in \mathbb{R}^l$ and $\boldsymbol{v}_k^{(m)} \in \mathbb{R}^l$, respectively. Here $m = [1, M] \in \mathbb{N}_+$ is the number of the particle, and $k = 1, 2, \dots,$ is a discrete point in time. These are updated as follows.

Given ω, c_1, c_2 are specified, $\boldsymbol{z}_0^{(m)}$ and $\boldsymbol{v}_0^{(m)}$ are set using uniform random numbers. The algorithm starts from Step1 1 at $k = 1$.

Step 1 The optimal solutions for every particle at the previous instant $\hat{\boldsymbol{z}}_{k-1}^{(m)}$ are re-evaluated at the current instant and the lowest value is found using

$$\tilde{\boldsymbol{z}}_k^g = \arg\min \left\{ f_k(\hat{\boldsymbol{z}}_{k-1}^{(m)}) \right\} \quad (21)$$

Step 2 The velocity and position of every particle are updated.

$$\boldsymbol{v}_k^{(m)} = \eta \boldsymbol{v}_{k-1}^{(m)} + c_1 r_1 \left(\hat{\boldsymbol{z}}_{k-1}^{(m)} - \boldsymbol{z}_{k-1}^{(m)}\right) + c_2 r_2 \left(\tilde{\boldsymbol{z}}_{k-1}^{g} - \boldsymbol{z}_{k-1}^{(m)}\right)$$

$$\boldsymbol{z}_k^{(m)} = \boldsymbol{z}_{k-1}^{(m)} + \boldsymbol{v}_k^{(m)}$$

Step 3 The position of every particle producing the lowest fitness value is saved together with the value of $f_k(\hat{\boldsymbol{z}}_k^{(m)})$

$$\hat{\boldsymbol{z}}_k^{(m)} = \begin{cases} \boldsymbol{z}_k^{(m)} & (\text{if } f_k\left(\boldsymbol{z}_k^{(m)}\right) < f_k\left(\hat{\boldsymbol{z}}_{k-1}^{(m)}\right)) \\ \hat{\boldsymbol{z}}_{k-1}^{(m)} & (\text{otherwise}) \end{cases}$$

Step 4 The best solution in the entire swarm is found:

$$\hat{\boldsymbol{z}}_k^g = \arg\min \left\{ f_k\left(\hat{\boldsymbol{z}}_k^{(m)}\right) \right\}$$

This is taken as the OPSO estimate at time k.

Step 5 The algorithm returns to Step 1 with $k = k + 1$

C. Region of Investigation

Here, the search ranges of the values of N and σ are described. The value of N is required to be higher than 21; the upper limit of which is not set. Both ends of the discretely approximated test function are required to tend towards 0, and thus, the value of the variance is set as follows:

$$3.3\sigma \leq \frac{NT_s}{2} \quad (22)$$

The relationship between the test function and the variance is shown in Fig. 1.

IV. Simulation

A. Target Signal and Parameters Setting

The following target signal is used for evaluating the proposed method.

$$x(t) = \begin{cases} \cos 5\pi k T_s & (\text{if } t \leq 0.2 \text{ [s]}) \\ -\cos 20\pi k T_s & (\text{otherwise}) \end{cases}$$

Here, $T_s = 0.001$ [s] and its time evolution is shown in Fig. 2. The angular frequency is estimated by the FED and the values of N and σ, given by OPSO, are optimized. Here, the number of particles for OPSO is set as 100, the value of the parameter $\alpha = 0.001$ is shown in III-A, and the parameters of OPSO are set as $\eta = 0.4$, $c_1 = 9$, and $c_2 = 1$. At the initial time, the particles were distributed at random $N \in [21, 100]$ and $\sigma \in (0, 0.015]$.

B. Results

The time evolution of the particle distribution and the global best particle are shown in Fig. 3. At time $k = 50$, as shown in Fig. 3(a), the particles were distributed in the search region, and the global best was not arrived at in correspondence to the best solution. At time $k = 200$ and $k = 400$, as shown in Fig. 3(b) and (c), the distribution of particles converged and the global best was approximately the best solution. The time evolution of the searched values of N and σ by OPSO is shown in Fig. 4. The values increased until 0.15 [s] with fluctuations. After a change in the angular frequency at $t = 0.2$ [s], the values almost converged after 0.3 [s]. The searched value of N decreased slightly after 0.2 [s]. The cause of this reduction is that it was better even with a small value

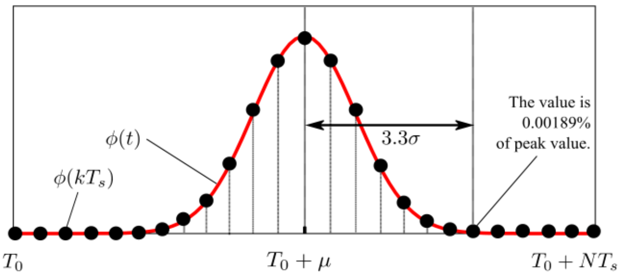


Fig. 1. Test function and search space for σ.

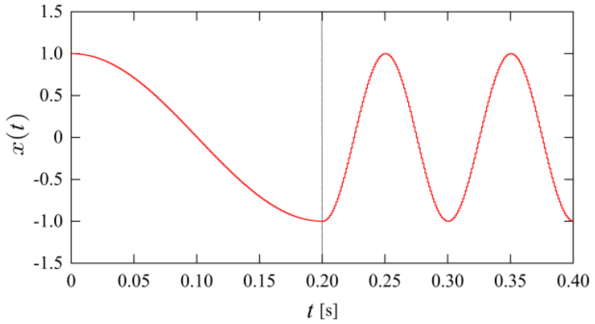
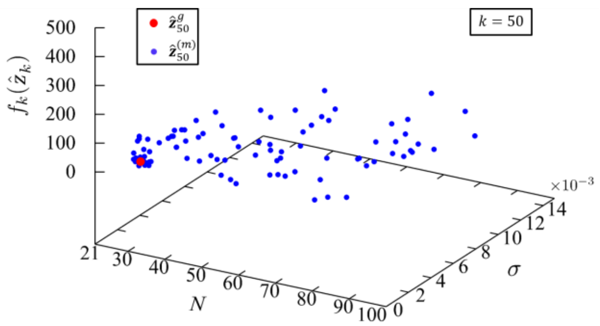
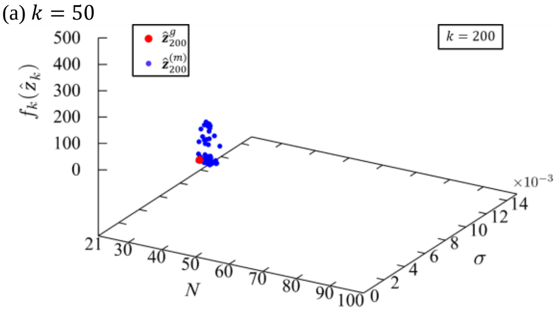


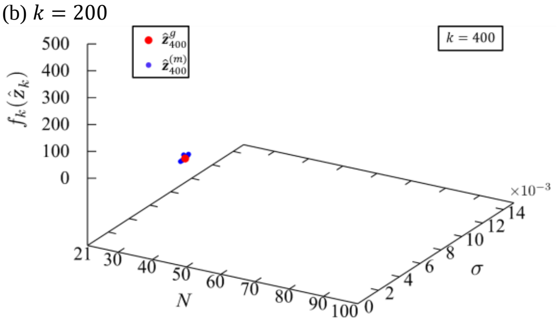
Fig. 2 Target signal with angular frequency change at 0.2 [s].



(a) $k = 50$



(b) $k = 200$



(c) $k = 400$

Fig.3 The particle distribution.

of N by increasing the angular frequency of the signal. The values at $t = 0.4$ [s] were $N = 40$ and $\sigma = 0.003882$. In this case, the maximum value of σ in Eqn. (22) was 0.00606, and the best value was found to be approximately 64 % of this value. The values of the cost function $f_k(\hat{\mathbf{z}}_k^g)$ and the relative error in the estimation and grand truth are shown in Fig. 5. The values converged at 0.2 [s] with fluctuations, beyond which, the values increased after a change in angular frequency. The cause of this phenomenon is that two angular frequencies were mixed during N sampling. It was found from Fig. 5 that the

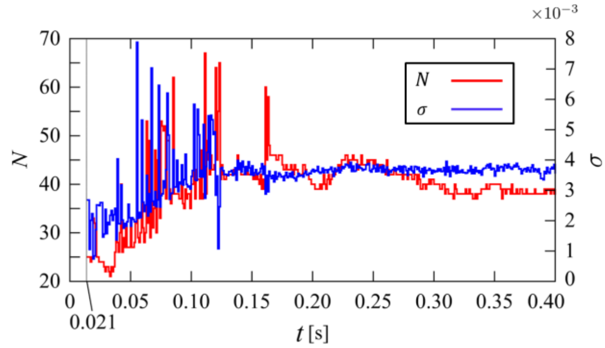


Fig.4 Time evolution of the derived N and σ.

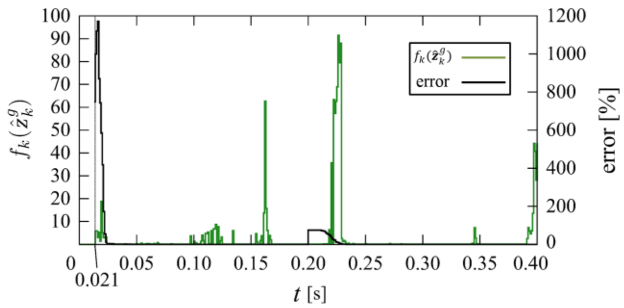


Fig. 5 Time evolution of the value of cost function and the estimated error using calculated values of N and σ.

relative estimation error was maintained under 0.033% by using the parameters obtained from the OPSO.

V. Conclusion

In this paper, we first described the outline of the FED, and explained that this method has a certain degree of freedom in the method of determining two parameters: N and σ. Next, we proposed a method to adjust these values adaptively by using the OPSO algorithm for a time-varying signal. The estimation accuracy and the calculation time of the FED are optimized by the proposed method. Moreover, it was found from the results of the numerical simulation that high estimation accuracies could be obtained by using optimized parameters. In future work, we will implement the proposed method into an actual system to verify its validity.

References

[1] A. Ishizaka, M. Nitta, K. Koto: "Error analysis on distribution-based frequency estimator", Proc. of Int. Conf. on Control, Automation, and Systems, pp. 255–260, 2010.

[2] J. Kennedy and R. C. Eberhart: "Particle Swarm Optimization", Proc. of IEEE Int. Conf. on Neural Network IV, pp.1942–1948, 1995.

[3] T. Nishida and T. Sakamoto: "Adaptive PSO for Online Identification of Time-varying Systems", Electronics and Communications in Japan, Vol. 95, Iss. 7, pp. 10–18, 2012.