Application of Competitive Associative Nets to Plane Extraction from Range Data

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Abstract. This article describes an application of competitive associative net called CAN2 to plane extraction from 3D range images measured by a laser range finder (LRF). The CAN2 basically is a neural net which learns efficient piecewise linear approximation of nonlinear functions, and in this application it is utilized for learning piecewise planner (linear) surfaces from the range data. As a result of the learning, the obtained piecewise planner surfaces are more precise than the actual planner surfaces, so that we introduce a method to gather piecewise planner surfaces for reconstructing the actual planner surfaces. We apply the method to the real range data, and examine the effectiveness and the performance.

1 Introduction

In the recent decade, we are developing competitive associative net called CAN2 which is a neural net utilizing competitive and associative schemes [1, 2] for learning piecewise linear approximation of nonlinear functions. The effectiveness in many applications such as function approximation, control, rainfall estimation, time-series prediction, etc. has been shown so far (see [3]-[11]). For example, our method using the CAN2 was awarded the regression winner at Evaluating Predictive Uncertainty Challenge held at NIPS2004 [10].

So, recently in [11], we began to apply the CAN2 to 3D range data or range images for taking advantages of the abilities of the CAN2 such as piecewise linear approximation, noise reduction, data compression, and so on. As range images, we use the ones obtained by a laser range finder (LRF) which can measure with high-speed and high-precision relatively to stereo-vision methods without affected by environmental light. However, it has several disadvantages, such as (1) the images sometimes involve lack of data called black spots, (2) the range data involve quantization errors owing to the depth resolution (e.g. 10[mm] ), and (3) the number of data in a range image is large due to high angular resolution (e.g. 0.25[deg] ). In [11], we have coped with the above problems and tried to improve the approximation performance. In this article, we try to apply the CAN2 to extracting planes in the range images. Since the CAN2 learns to extract piecewise planner (linear) surfaces of the range data more precisely than the real planner surfaces, we present a method to gather piecewise planner surfaces for reconstructing the actual planner surfaces. Here, note that there are several researches for extracting planes from range data, using several approaches such as
Hough transform [13] and expectation maximization (EM) paradigm [14]. The researches are mainly for mobile robots to generate 3D environmental models of walls, ceilings, building facades, and so on. In this article, we use the range images of much smaller objects such as a small cubic box, an extinguisher, and so on.

In the next section, we briefly describe the CAN2 and the range image to be processed, and then present a method for extracting planes in a range image. In Sec.3, we show several experimental results, and examine the effectiveness of the present method.

2 A Method to Extract Planes from Range Image

2.1 CAN2 for Piecewise Linear Approximation

For introducing the CAN2, we first consider an input-output system with a $k$-dimensional input vector $\mathbf{x}^s_j = (x^s_{j1}, x^s_{j2}, \cdots, x^s_{jk})^T$ and a scalar output $y^s_j$ given by

$$y^s_j = f(\mathbf{x}^s_j) + d_j,$$

where $j = 1, 2, \cdots$ indicates the index to distinguish the data, $f(\mathbf{x}^s_j)$ is a non-linear function of $\mathbf{x}^s_j$, and $d_j$ represents zero-mean noise with the variance $\sigma^2_d$. The superscript “$s$” indicate the variables related to the input-output system for distinguishing the positions such as $x_{mn}$ and $y_{mn}$ described below. We use $k = 2$ for the application to the 3D range data shown below. A CAN2 has $N$ units (see Fig.1), and the $i$th ($i \in I = \{1, 2, \cdots, N\}$) unit has a weight (column) vector $\mathbf{w}_i^s = (w_{i1}, \cdots, w_{ik})^T \in \mathbb{R}^{k \times 1}$ and an associative matrix (or a row vector) $\mathbf{M}_i = (M_{i0}, M_{i1}, M_{i2}) \in \mathbb{R}^{1 \times (k+1)}$. The CAN2 approximates the above function $y^s = f(\mathbf{x}^s)$ by

$$\hat{y}^s = \hat{y}^c = \mathbf{M}_c \hat{\mathbf{x}}^c,$$

where $\hat{\mathbf{x}}^c = (1, (\mathbf{x}^s)^T)^T$, and the $c$th unit is the winner of the following minimization or competition,

$$c \triangleq \arg\min_{i \in I} \|\mathbf{x}^s - \mathbf{w}_i\|.$$

Note that the above function approximation partitions the input space $V = \mathbb{R}^k$ into $N$ Voronoi regions

$$V_i \triangleq \{\mathbf{x}^s \mid i = \arg\min_{j \in I} \|\mathbf{x}^s - \mathbf{w}_j\\}$$

for $i \in I$, and performs piecewise linear approximation of $y^s = f(\mathbf{x}^s)$. Further, we have developed an efficient batch learning method (see [7] for details), and in the following we suppose to use the CAN2 after the batch learning.
2.2 Range Image

We use a SICK LMS200 laser range finder (LRF) for scanning the horizontal 2D plane to measure the distance to an object, where the maximum measurable distance is 8191[mm] with the resolution 10[mm], and the scanning range is 180[deg] with the angular resolution 0.25[deg]. In order to scan 3D space, we have designed and made a suspension unit [12] for rotating the LRF vertically by means of a geared stepping motor with an angle resolution 0.05[deg] (Fig. 2). So that we can obtain the polar range data \((\theta_m, \phi_n, d_{mn})\) consisting of the horizontal angle \(\theta_m\), the vertical angle \(\phi_n\) and the measured distance \(d_{mn}\). Further, the 3D rectangular (Cartesian) position \(x_{mn} = (x_{mn}, y_{mn}, z_{mn})\) consisting of the horizontal, vertical and depth positions, \(x_{mn}, y_{mn}\) and \(z_{mn}\), respectively, can be obtained by

\[
x_{mn} = \begin{pmatrix} x_{mn} \\ y_{mn} \\ z_{mn} \end{pmatrix} = \begin{pmatrix} d_{mn} \sin \theta_m \\ d_{mn} \cos \theta_m \sin \phi_n \\ d_{mn} \cos \theta_m \cos \phi_n \end{pmatrix}.
\]  

(5)

Actually, we use the range image in the polar form given as

\[
D_{\text{polar}} = \{(\theta_m, \phi_n, d_{mn}) | \theta_m = 0.25m[\text{deg}], m_{\text{min}} \leq m \leq m_{\text{max}}, \phi_n = 0.25n[\text{deg}], n_{\text{min}} \leq n \leq n_{\text{max}}\},
\]  

(6)
where \( m \) and \( n \) are integers, and transform it to the one in the rectangular form given as

\[
D_{\text{rect}} = \{(x_{mn}, y_{mn}, z_{mn}) | x_{\min} \leq x_{mn} \leq x_{\max}, y_{\min} \leq y_{mn} \leq y_{\max}\}
\] (7)

via Eq.(5), where \( x_{\min}, x_{\max}, y_{\min} \) and \( y_{\max} \) indicate the range of the width and the height which contains a target object to be processed.

2.3 Extracting Planes via the CAN2

Suppose we train the CAN2 with the data \( x_{mn} = (x_{mn}, y_{mn}, z_{mn}) \) in the rectangular range image \( D_{\text{rect}} \), where the relation \( z_{mn} = f(x_{mn}, y_{mn}) + d_{mn} \) is supposed to be fulfilled as the input-output system given by Eq.(1) with \( y_s^* = z_{mn} \) and \( x_s^* = (x_{mn}, y_{mn})^T \). Further, we suppose that the range image consists of several planes (plane segments or planner surfaces) \( P_l \) for \( l = 1, 2, \ldots \), and the CAN2 after the training approximates each \( P_l \) by means of piecewise linear (planner) surfaces \( P_{\text{CAN2}}^i \) for \( i \in J_l \). Precisely, the \( i \)th unit of the CAN2 with \( \mathbf{w}_i = (w_{i1}, w_{i2})^T \) and \( M_i = (M_{i0}, M_{i1}, M_{i2}) \) approximates the distance \( z (= \hat{y}^*) \) for \( (x, y)^T \) (or \( x^* = M_i \hat{x}^* \)) in the Voronoi region \( V_i \) with the centroid \( \mathbf{w}_i \) by

\[
z = M_{i0} + M_{i1}x + M_{i2}y \quad \text{for} \ (x, y)^T \in V_i.
\] (8)

Here, note that this plane is also represented with the unit normal vector \( \mathbf{n}_i = (n_{ix}, n_{iy}, n_{iz})^T \) and the distance \( \alpha_i \) from the origin of the coordinate system as

\[
n_{ix}x + n_{iy}y + n_{iz}z - \alpha_i = 0.
\] (9)

Thus, we have the relation between the unit normal vector \( \mathbf{n}_i \) and the associative matrix \( M_i \) as

\[
(\mathbf{n}_i^T, \alpha_i) = \frac{(-M_{i1}, -M_{i2}, 1, M_{i0})}{\sqrt{M_{i1}^2 + M_{i2}^2 + 1}}
\] (10)
Fig. 3. Relation of the plane segment $P_i$ and the piecewise plane $P_{i}^{\text{CAN2}}$. If $P_i$ and $P_{i}^{\text{CAN2}}$ are on the same plane with an allowable small error, (1) the angle $\cos^{-1}(\bar{n}_i^T n_i)$ between the normal vectors is small, (2) the angle $\theta_i$ between $\bar{u}_i$ and $(\bar{u}_i - u_i)$ is almost 90[deg], and (3) the angle $\theta_i$ between $n_i$ and $(\bar{u}_i - u_i)$ is almost 90[deg].

and

$$
M_i = \left( \begin{array}{c}
\frac{n_{ix} - n_{iy}}{n_{iz}} - \frac{n_{iy}}{n_{iz}} \frac{\alpha_i}{n_{ix}} \\
\frac{\alpha_i}{n_{iz}} \end{array} \right).
$$

(11)

Here, note that $M_i$, which determines $P_{i}^{\text{CAN2}}$ via Eq.(8), is the product of learning the range image involving noise, so that $P_{i}^{\text{CAN2}}$ and $P_{j}^{\text{CAN2}}$ may be (slightly) different even if they approximate the parts of the same plane $P_i$. Thus, we identify the plane $P_i$ as the mean of piecewise planes $P_{i}^{\text{CAN2}}$ for $i \in J_l$, where $J_l$ denotes the set of indices $i$ of $P_{i}^{\text{CAN2}}$ which approximate the same plane $P_i$. Further, let $\bar{w}_l$ and $\overline{M}_l$ denote the mean of $w_i$ and $M_i$ for $i \in J_l$, respectively, and let $u_i$ be the 3D vector corresponding to the weight vector $w_i$, i.e., $u_i = (\bar{w}_i^T, M_i \bar{w}_i)$ where $\bar{w}_i = (1, \bar{w}_i^T)^T$, and so does $\bar{u}_l = (\bar{w}_l^T, \overline{M}_l \bar{w}_l)^T$ (see Fig. 3). Then, we obtain $J_l$ and $P_l$ for $l = 1, 2, \cdots$ consecutively from $P_{i}^{\text{CAN2}}$ ($i = 1, 2, \cdots, N$) by means of the following procedure.

**Step 1:** Set $l := 1$, $\bar{w}_l := w_1$, $\overline{M}_l := M_1$, $I_l := \{1, 2, \cdots, N\}$, where “:=” denotes substitution, and $I_l$ indicates the set of indices $i$ of $P_{i}^{\text{CAN2}}$ from which the elements of $J_l$ are obtained.

**Step 2:** If $I_l$ is empty, quit. Otherwise, obtain $J_l$ consisting of the indices $i$ which are in $I_l$ and satisfy the following conditions:

$$
|\bar{n}_l^T n_i| \geq \cos \theta_c,
$$

(12)

$$
\frac{|\bar{n}_l^T (\bar{u}_l - u_i)|}{||\bar{u}_l - u_i||} \leq \sin \theta_c,
$$

(13)

$$
|n_i^T (\bar{u}_l - u_i)| \leq \sin \theta_c,
$$

(14)

where $\theta_c$ is a small positive constant indicating a threshold, and the unit normal vectors $n_i$ and $\bar{n}_l$ of $P_{i}^{\text{CAN2}}$ and $P_l$, respectively, are selected from
Eq.(10) with $M_i$ and $\overline{M}_i$. Note that Eqs. (12), (13) and (14), respectively, correspond to the conditions (1), (2) and (3) described in Fig. 3.

**Step 3:** Update $\overline{w}_t$ and $\overline{n}_t$ as

$$\overline{w}_t := \frac{1}{|J_t|} \sum_{i \in J_t} w_i, \quad (15)$$

$$\overline{M}_t := \frac{1}{|J_t|} \sum_{i \in J_t} M_i, \quad (16)$$

where $|J_t|$ indicates the number of elements in $J_t$.

**Step 4:** Repeat **Step 2** and **Step 3** until $\overline{w}_t$ and $\overline{M}_t$ converge.

**Step 5:** Set $I_{t+1} := I_t \setminus J_t$, $l := l + 1$, and go to **Step 2**.

Here, we would like to note that we also tried to use $\overline{n}_t = \sum_{i \in J_t} n_i$ instead of Eq.(16) and Eq.(10) for obtaining $\overline{n}_t$, but it did not work well.

### 3 Experiments and Remarks

We have examined the present method for the range images of several objects, where the arrangement of the LRF and the cube is as shown in Fig. 4.

First, we show the result of the present method applied to the range image of a cube as shown in Fig. 5. The range image in the rectangular form is shown in Fig. 5(a), where the image consists of 15647 (three-dimensional) data obtained from the polar range data consisting of about 16000 data in the range of $25[\text{deg}]$ (width)$ \times 40[\text{deg}]$ (height) with the resolution 0.25[deg]. The CAN2 with 400 units after 5 batch learning iterations approximates the image as shown in Fig. 5(b), where 50 $\times$ 50 lattice grids on the $xy$-plane are used for representing the image. Since the CAN2 requires the memory for 400 units of 2-dimensional $w_i$ and 3-dimensional $M_i$, the memory compression ratio is about 0.04 ($\approx (400 \times (2 + 3))/(15647 \times 3)$). There are seven plane segments in the range image, and we could have obtained all of them as shown in Fig. 5(c) and (d), where we have used the angle threshold $\theta_c = 20[\text{deg}]$ (see **Step 2**), and repeated 10 times of the repetition for obtaining a convergence (see **Step 4**). Although
Fig. 5. Result of extracting planes from the range data of a cube on a floor in front of a wall. (a) shows the range image in the rectangular form, (b) is the approximated range image for 50×50 lattice grids on xy-plane, where the approximation is done via the CAN2 with 400 units after 5 batch learning iterations. The different marks in (c) show the different plane segments for the 50×50 lattice grids as shown in (b), while the marks in (d) show the plane segments for the centroid vector $u_i$ of the piecewise planes $P_i^{CAN2}$. The thick lines in (d) show the intersection of the obtained plane segments.

The boundary of the obtained plane segments shown in Fig. 5(c) are not so sharp, we can obtain the sharp boundary as shown in Fig. 5(d) by means of calculating the intersection of the obtained plane segments.

Here, the threshold $\theta_t = 20[\text{deg}]$, which is obtained as a result of trial and error, seems to be big because it indicates that two planes with crossing angle less than about 20[deg] cannot be discriminated. Although there may be methods to overcome this problem, we here only show the difficulty of the problem by means of showing the result of the present method applied to the wall range image. Namely, the range image in the rectangular form and the approximated range image are shown in Fig. 6(a) and (b), where we can see the wrinkled approximated image. Further, Fig. 6(c) shows the range of the measurement errors are more than 10[mm], and Fig. 6(d) of the original range image in
Fig. 6. Result of extracting a plane from the range data of a wall. (a) shows the range image in the rectangular form and (b) is the approximated range image viewed from the same direction as for (a), where the CAN2 for the approximation is with 20 units and applied 5 batch learning iterations. (c) depicts the range data shown in (a) viewed from the lefthand side of the wall. The thick lines in (a), (b) and (c) indicate the edges of the obtained same plane segment. (d) shows the original range image in the polar form.

The polar form indicates the wrinkled errors and their errors are caused by the measurement depth resolution about 10[mm] which is low relatively to the angular resolution 0.25[deg] of the angles $\theta$ and $\phi$ which correspond to the $x$ and $y$ rectangular positions whose resolution around $z = 1040[mm]$ from the origin is about $4.5(\approx 1040 \tan(0.25\pi/180))[mm]$ while the distance discriminable from $1040[mm]$ with the resolution 10[mm] is $1050[mm]$ and corresponds to $145(\approx \sqrt{1050^2 - 1040^2})[mm]$ in the $x$ or $y$ direction. From the wrinkled range image as shown in Fig. 6(b), we can see the range of the orientation of the tangent planes of the wrinkles is not small, and we have to set $\theta_2$ big enough for them to be the same plane segment.

As a more complicated range image, we have applied the present method to a range image of an extinguisher. The range image shown in Fig. 7(a) consists of 3-dimensional 20115 range data, while the one shown in Fig. 7(b) requires the CAN2 with 400 units of 2-dimensional $w_i$ and 3-dimensional $M_i$, so that the compression ratio is about 0.03 (\(\approx 400 \times (2 + 3)/(20115 \times 3)\)). Although we
Fig. 7. Result of extracting planes from the range image of an extinguisher in front of a wall. (a) shows the range image in the rectangular form, (b) is the approximated range image for $50 \times 50$ lattice grids on $(x, y)$ plane obtained, where the CAN2 with 400 units after 5 batch learning iterations is used. The different marks on $50 \times 50$ lattice grids in (c) show the different plane segments obtained by means of the present method, while the marks in (d) show the plane segments for the centroid vector $u_i$ of the piecewise planes $P_{CAN2}^i$. The thick lines in (d) show the intersection of obtained major 7 plane segments.

We can see a certain level of approximation performance, we hope that the CAN2 learns more precise structure of the extinguisher, which is for our future work. The extracted planes are shown in Fig. 7(c) and (d), where the wall behind the extinguisher is successfully extracted and the complicated surfaces of the extinguisher are approximated by several planes surrounding the extinguisher.

4 Conclusion

We have presented a method to extract planes from a range image. Since the CAN2 learns to extract piecewise planner surfaces of the range image more precisely than the real planner surfaces, the method is designed to gather piecewise planner surfaces for reconstructing the real planner surfaces. The present method successfully extract all visible planner surfaces of a cube and several
approximated surfaces surrounding an extinguisher as well as their background walls. However, it could not extract more precise surfaces of the extinguisher, which is owing mainly that the parameter $\theta_i$ of the present method indicating the distinguishable crossing angle of surfaces could not be set so small because a real flat wall is wrinkled in the range image in the rectangular form due to the measurement resolution of the LRF. Further, the CAN2 could not learn precise structure of the extinguisher, while the improvement may conflict with the removal of the above wrinkles in the range image. We would like to overcome these problems in our future research.

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References