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State estimation and control of a mobile robot using Kalman filter and particle filter

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Abstract: We focus on the filter in a method proposed recently, i.e., a method to improve the state estimation accuracy of mobile robots placed near high-rise buildings using the GPS (global positioning system) signals. In this method, it was assumed that a GPS signal that contains a reflected and diffracted wave is denoted by the sum of the true position information and noise that follows a time-varying Gaussian distribution. On the basis of this assumption, the time-varying bias of a GPS signal is tracked using a Kalman filter. In addition, a particle filter, which executes sampling and likelihood evaluation using the estimated bias, was developed. With the proposed method, a GPS signal that contains the rejected noise introduced by the conventional method can be used efficiently, and the state estimation accuracy of the robot in a shadow area of GPS satellite can be improved. In this research, the control system for an autonomous mobile robot incorporating the proposed state estimation mechanism is evaluated via different simulation condition from the previous work.

Keywords: GPS, mobile robot, Kalman filter, particle filter.

1 INTRODUCTION

Theoretically, if there are four or more GPS (global positioning system) satellites that can carry out signal reception directly using an antenna, position measurement is possible [1]; the positioning quality can be improved via reception from more visible satellites [2]. In general, the GPS satellite that can carry out signal reception directly is called a *visible satellite* and other satellites are called *invisible satellites*. Moreover, an area where a required number of visible satellites are not available is called a *shadow area*. The measurement error of position becomes significant because shadow areas occur where many high-rise buildings exist around a mobile robot which equips GPS antenna [1].

Recently, we proposed a method to improve the state estimation accuracy of mobile robots placed near high-rise buildings using the statistical property of the reflected and diffracted waves of GPS signals [3]. First, it was assumed that a GPS signal that contains reflected and diffracted waves is represented by true position information and the sum of the noise that follows a time-varying Gaussian distribution. Based on this assumption, the time-varying bias of the Gaussian distribution is estimated using a KF (Kalman filter), and a PF (particle filter), which executes sampling and likelihood evaluation using the estimated bias, are developed. According to this state estimation mechanism, a GPS signal was effectively used and the state estimation accuracy of a robot in the shadow area was improved. Furthermore, the control system of an autonomous mobile robot incorporating the proposed state estimation method was developed, and its effectiveness was evaluated via simulation. In this paper, first, this method is explained, and then, is evaluated using different conditions from the previous work.

2 PROBABILISTIC MODEL

2.1 Kinematic model

Let a front-wheel steering rear-wheel drive type four wheel mobile robot be a controlled object in this paper. The robot's state variables are defined by the position and angle in the inertial reference system $O-XY$:

$$\boldsymbol{x}_k \triangleq [x_k \quad y_k \quad \theta_k]^T = [\boldsymbol{z}_k^T \quad \theta_k]^T \tag{1}$$

where $k = 0, 1, 2, \cdots$ is discrete time, $\boldsymbol{z}_k^T \triangleq [x_k \quad y_k]^T$ is the center position of the robot based on the center the rear wheel axle, θ_k is the angle between the robot's direction of movement and the X axis, and l is the length of the wheel base. Next, the motion reference input is defined as $\boldsymbol{u}_k \triangleq [v_k \quad \phi_k]^T$, where v_k denote the direction of movement speed, which occurs because the driving wheel and ϕ_k denotes the steering angle of the front wheel, which makes a positive counterclockwise rotation. The discrete time state equation of the mobile robot can be expressed as follows [4]:

$$\boldsymbol{x}_{k+1} = f(\boldsymbol{x}_k, \boldsymbol{u}_k) = \boldsymbol{x}_k + \begin{bmatrix} \frac{v_k}{\omega_k}\{\sin(\theta_k + \omega_k\Delta) - \sin\theta_k\} \\ -\frac{v_k}{\omega_k}\{\cos(\theta_k + \omega_k\Delta) - \cos\theta_k\} \\ \omega_k\Delta \end{bmatrix}, \tag{2}$$

where $\omega_k \triangleq (v_k/l)\tan\phi_k$ and Δ is the sampling time.

2.2 Probabilistic discrete state space model

In an actual robot system, some noise may merge with the control input under the influence of backlash/degradation of the drive system or an uneven road surface, etc. Therefore, we assume that the disturbance according to a normal distribution merges with the command input and the control input is modeled as follows:

$$\hat{v}_k \triangleq v_k + \varepsilon_v, \ \ \varepsilon_v \backsim \mathcal{N}(0, \alpha_1 v_k^2 + \alpha_2 \phi_k^2) \tag{3}$$

$$\hat{\phi}_k \triangleq \phi_k + \varepsilon_\phi, \ \ \varepsilon_\phi \backsim \mathcal{N}(0, \alpha_3 v_k^2 + \alpha_4 \phi_k^2) \tag{4}$$

where $\alpha_i\ (i = 1,2,3,4)$ is a positive constant that determines the characteristic of the noise. In the similar manner, considering the disturbance added in the direction of the robot by road surface unevenness etc., the transition of the angle of direction is modeled as follows:

$$\hat{\theta}_{k+1} = \hat{\theta}_k + \left(\widehat{w}_k + \varepsilon_\gamma\right)\Delta, \varepsilon_\gamma \sim \mathcal{N}(0, \alpha_5 v_k^2 + \alpha_6 \phi_k^2), \quad (5)$$

where $\widehat{w}_k \triangleq (\hat{v}_k/l)\tan\hat{\phi}_k$ and $\alpha_j\ (j = 5,6)$ is the positive constant that determines the characteristic of the noise. For simplification, the noises are described as $\boldsymbol{\varepsilon}_k \triangleq [\varepsilon_v \quad \varepsilon_\phi \quad \varepsilon_\gamma]^T$ denotes the state variables containing the probabilistic elements.

From the above relations, Eq. (3) can be described as a probabilistic discrete state model as follows:

$$\hat{\boldsymbol{x}}_{k+1} \sim p_f(\hat{\boldsymbol{x}}_{k+1}|\hat{\boldsymbol{x}}_k, \boldsymbol{u}_k) = \hat{\boldsymbol{x}}_k + \begin{bmatrix} \frac{\hat{v}_k}{\widehat{\omega}_k}\{\sin(\hat{\theta}_k + \widehat{\omega}_k\Delta) - \sin\hat{\theta}_k\} \\ -\frac{\hat{v}_k}{\widehat{\omega}_k}\{\cos(\hat{\theta}_k + \widehat{\omega}_k\Delta) - \cos\hat{\theta}_k\} \\ \widehat{\omega} + \varepsilon_{\gamma_k}\Delta \end{bmatrix}, \quad (6)$$

where $\hat{\boldsymbol{x}}_k \triangleq [\hat{x}_k \quad \hat{y}_k \quad \hat{\theta}_k]^T$ denotes the state variables containing the probabilistic element.

2.3 Observation model

The mobile robot observes the self-position $\boldsymbol{z}_k^{\text{GPS}} \triangleq [x_k^{\text{GPS}} \quad y_k^{\text{GPS}}]^T$ at each discrete time interval using the provided GPS antenna as $\boldsymbol{z}_k^{\text{GPS}} = \boldsymbol{z}_k + \boldsymbol{\xi}_k$, where

$$\boldsymbol{\xi}_k \sim \begin{cases} \mathcal{N}(r_k, \boldsymbol{\Sigma}_s) & \text{if in a shadow area} \\ \mathcal{N}(0, \boldsymbol{\Sigma}_b) & \text{otherwise.} \end{cases} \quad (7)$$

Thus, $\boldsymbol{\xi}_k$ is a noise vector according to the time-varying normal distribution from which its bias and variance change with respect to the environment around of the robot. Here, the covariance matrices are $\boldsymbol{\Sigma}_s = \text{diag}[\sigma_s^2, \sigma_s^2]$ and $\boldsymbol{\Sigma}_b = \text{diag}[\sigma_b^2, \sigma_b^2]$. In general, because the distribution in the shadow area becomes larger than that of a direct wave, the condition $\det(\boldsymbol{\Sigma}_b) < \det(\boldsymbol{\Sigma}_s)$ is fulfilled.

Next, the measurement of direction is not included in a GPS signal, and therefore, it is calculated from the robot's direction as follows:

$$\theta_k^{\text{GPS}} = \tan^{-1}\left(\frac{y_k^{\text{GPS}} - \hat{y}_{k-1}}{x_k^{\text{GPS}} - \hat{x}_{k-1}}\right) + w_{k-1}\Delta. \quad (8)$$

An observation vector consists of these observed values as follows:

$$\boldsymbol{h}(\hat{\boldsymbol{x}}_k) = \boldsymbol{y}_k^{\text{GPS}} \triangleq [x_k^{\text{GPS}} \quad y_k^{\text{GPS}} \quad \theta_k^{\text{GPS}}]^T. \quad (9)$$

3 STATE ESTIMATION AND CONTROL

3.1 Control system including particle filter

The state variable diagram of the robot control system developed in this research is shown in Fig. 1. This control system is developed for the robot which passed through the specified domain by controlling only the steering angle under the speed regularity of the robot. In this control system, the state transition of the robot is represented using a nonlinear model and the noise according to the time-varying Gaussian distribution is added to the observed signal.

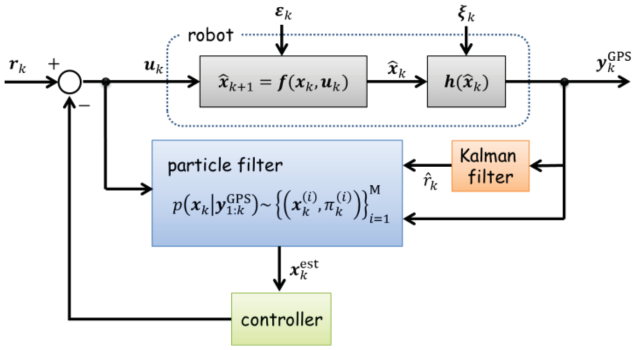


Fig. 1. State variable diagram with Kalman filter and particle filter.

When the changes in the bias and variance of the noise in the GPS signal are large, in order to continue driving a control system stably, it is necessary to include the statistical characteristics of the GPS signal in the shadow area in a state estimator as prior knowledge. Then, in order to estimate the bias of the GPS signal, first, a KF is employed. Next, the state estimation of the robot is executed using the bias estimated by the KF, and the PF incorporates the statistical characteristics of the noise of the GPS signal as prior knowledge. Then, the control input is calculated via PF using the estimation.

3.2 Bias estimation by Kalman filter

The output is represented as a system including the noise following a Gaussian distribution as follows:

$$\boldsymbol{\eta}_{k+1} = \boldsymbol{A}\boldsymbol{\eta}_k \quad (10)$$

$$\boldsymbol{z}_k^{\text{GPS}} = \boldsymbol{C}\boldsymbol{\eta}_k + \boldsymbol{\omega}_k \quad (11)$$

where

$$\boldsymbol{\eta}_k \triangleq \begin{bmatrix} \hat{x}_k \\ \hat{v}_{xk} \\ \hat{y}_k \\ \hat{v}_{yk} \end{bmatrix}, \quad \boldsymbol{A} \triangleq \begin{bmatrix} 1 & \Delta & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \boldsymbol{C} \triangleq \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad (12)$$

$$\boldsymbol{\omega}_k \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{\Sigma}_\omega), \quad \boldsymbol{\Sigma}_\omega \triangleq \text{diag}[\sigma_c^2, \sigma_c^2], \quad (13)$$

and $\hat{v}_{xk}$ and $\hat{v}_{yk}$ are velocities of the X- and Y-axis orientations, respectively, and $\boldsymbol{\Sigma}_\omega$ is a covariance matrix of $\boldsymbol{\omega}_k$. A KF is applied to this system to remove the noise according to a Gaussian distribution. Here, in order to take into consideration the noise according to a larger distribution, we set the variance as $\sigma_c^2 = \sigma_s^2$ based on the above assumption. The bias of a GPS signal the above KF as

$$\hat{r}_k = \sqrt{\hat{x}_k^2 + \hat{y}_k^2}. \quad (14)$$

3.3 Sampling

The PF executes state estimation using a set of M particles with weight $\{\boldsymbol{x}_k^{(i)}, \pi_k^{(i)}\}_{i=1}^M$. Here, $\boldsymbol{x}_k^{(i)}$ the position of the particles in the state space and $\pi_k^{(i)} \geq 0$ represents the weight of each particle. Subsequently, sampling, which is one of the estimation procedures using the PF, is described in detail.

In this research, the proposal distribution for movement of particles and distribution is designed as follows:

$$\tilde{\boldsymbol{x}}_k^{(i)} \sim p_q(\tilde{\boldsymbol{x}}_k^{(i)}|\boldsymbol{x}_{k-1}^{(i)}) = \begin{cases} p_f(\tilde{\boldsymbol{x}}_k^{(i)}|\boldsymbol{x}_{k-1}^{(i)}) & (95\% \text{ of particles}), \\ p_r(\tilde{\boldsymbol{x}}_k^{(i)}|\boldsymbol{x}_{k-1}^{(i)}) & (5\% \text{ of particles}). \end{cases} \quad (15)$$

In this procedure, function $p_f(\cdot)$ executes sampling for 95% of the particle set in consideration of the particle distribution, the previous input, and system noise. In constant, 5% of particles chosen at random are executed using sampling according to the function $p_r(\cdot)$ as shown below:

$$p_r(\boldsymbol{x}_k^{(i)}|\boldsymbol{x}_{k-1}^{(i)}) = \boldsymbol{f}(\boldsymbol{x}_{k-1}^{\text{est}}, \boldsymbol{u}_{k-1}^{(i)}) + \boldsymbol{R}_\psi \boldsymbol{\rho}(\hat{r}_k), \quad (16)$$

where $\boldsymbol{x}_{k-1}^{\text{est}}$ is the latest posteriori estimate,

$$\boldsymbol{\rho}(\hat{r}_k) \triangleq [\rho(\hat{r}_k) \quad 0]^T, \ \rho(\hat{r}_k) \sim \mathcal{N}(\hat{r}_k, \sigma_b^2), \quad (17)$$

$$\boldsymbol{R}_\psi \triangleq \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix}, \ \psi \in \text{rand}[-\pi, \pi], \quad (18)$$

Moreover, the weight of particles sampled by $p_q(\cdot)$ is given as follows:

$$\pi_k^{(i)} = \frac{1}{M\sqrt{2\pi\sigma_s^2}} \exp\left\{-\frac{1}{2}\frac{\rho(\hat{r}_k)^2}{\sigma_s^2}\right\}. \quad (19)$$

First, the particles are sampled on the basis of the state values estimated using the kinematic model according to the Gaussian distribution $\mathcal{N}(\hat{r}_k, \sigma_b^2)$. Then, random rotational coordinate transformations with radius $\hat{r}_k$ are executed on the particles. This is a contrivance to improve the accuracy of state estimation, and which enables it to use the GPS signal in the shadow area. The outline of these sampling methods is shown in Fig. 2.

3.4 Likelihood evaluation

The likelihood function based on the GPS signal in the area that receives the signals from a GPS satellite directly is given as follow:

$$p_{\text{direct}}\left(\boldsymbol{y}_k^{\text{GPS}}\middle|\boldsymbol{x}_k^{(i)}\right) = \frac{1}{3\pi\sigma_b^2}\left\{\exp\left(-\frac{d_x^2}{2\sigma_b^2}\right) + \exp\left(-\frac{d_y^2}{2\sigma_b^2}\right)\right\} + \frac{1}{6\pi\sigma_p^2}\exp\left(-\frac{d_\theta^2}{2\sigma_b^2}\right) \quad (20)$$

where $d_x = \delta(x_k^{(i)} - x_k^{\text{GPS}})$, $d_y = \delta(y_k^{(i)} - y_k^{\text{GPS}})$, $d_\theta = \delta(\theta_k^{(i)} - \theta_k^{\text{GPS}})$, and $\delta(\cdot)$ is the Dirac delta function. Furthermore, the likelihood function based on the GPS signal in the shadow area is given as follows:

$$p_{\text{shadow}}\left(\boldsymbol{y}_k^{\text{GPS}}\middle|\boldsymbol{x}_k^{(i)}\right) = \frac{1}{3\pi\sigma_s^2}\exp\left(-\frac{d_{xy}^2}{2\sigma_s^2}\right) + \frac{1}{6\pi\sigma_p^2}\exp\left(-\frac{d_\theta^2}{2\sigma_p^2}\right), \quad (21)$$

where $d_{xy} = \left|\delta(z_k^{(i)} - z_k^{\text{GPS}})\right| - \hat{r}_k$. The observation model of the PF is constructed by composition of these likelihood functions as follows:

$$p_h(\boldsymbol{y}_k^{\text{GPS}}|\boldsymbol{x}_k^{(i)}) = \frac{1}{2}p_{\text{direct}}(\boldsymbol{y}_k^{\text{GPS}}|\boldsymbol{x}_k^{(i)}) + \frac{1}{2}p_{\text{shadow}}(\boldsymbol{y}_k^{\text{GPS}}|\boldsymbol{x}_k^{(i)}). \quad (22)$$

The weight of particle is updated using this likelihood function as follows:

$$\tilde{\pi}_k^{(i)} \propto \pi_k^{(i)} p_h\left(\boldsymbol{y}_k^{\text{GPS}}\middle|\boldsymbol{x}_k^{(i)}\right) \quad (\forall i) \quad (23)$$

where all the weights are normalized so that $\sum_{i=1}^M \tilde{\pi}_k^{(i)} = 1$ after updating.

A posteriori estimation of the PF can be computed after

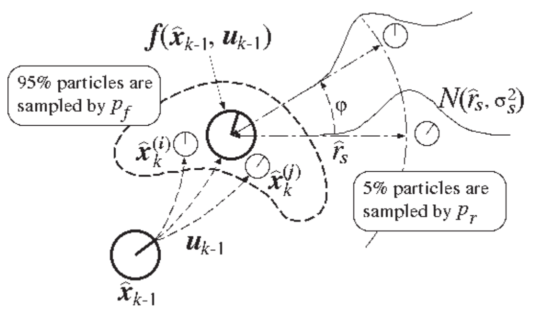


Fig. 2. Likelihood function of a measurement model.

likelihood evaluation as follow:

$$\boldsymbol{x}_k^{\text{est}} = \sum_{i=1}^M \pi_k^{(i)} \delta(\boldsymbol{x}_k^{(i)}). \quad (24)$$

Furthermore, a control input is drawn using this posteriori estimation as $\phi_{k+1} = k_\phi(\phi_k^* \ominus \phi_k)$, where $v_k = \text{const.}$, $\phi_k^* = \tan^{-1}\{(y_k^{\text{target}} - y_k^{\text{est}})/(x_k^{\text{target}} - x_k^{\text{est}})\}$, $\phi_k^*, \phi_k \in [0, 2\pi)$, $\ominus$ denotes the smallest angular difference on a circle, and k_ϕ denotes the feedback gain.

3.5 Resampling

Execution of resampling is dependent on the ESS (effective sample size):

$$ESS = \frac{1}{\sum_{i=1}^M \left(\tilde{\pi}_k^{(i)}\right)^2}. \quad (25)$$

$ESS = M$ when the weight of all the particles is equal and when the variety of weights is the largest, $ESS = 1$. This value is an indicator of the numger of particles currently utilized effectively and it is introduced in order to control the frequent ESS_{th} is prepared and resampling is executed if the obtained ESS is less than ESS_{th}. Resampling of $\boldsymbol{x}_k^{(i)}$ is carrried out by the probability of $\pi_k^{(i)}$ as follows:

$$\boldsymbol{x}_k^{(i)} \sim \begin{cases} \tilde{\boldsymbol{x}}_k^{(1)} & \text{with prob. } \tilde{\pi}_k^{(1)} \\ \vdots & \vdots \\ \tilde{\boldsymbol{x}}_k^{(M)} & \text{with prob. } \tilde{\pi}_k^{(M)} \end{cases}, \quad (\forall i) \quad (26)$$

Then the weights are normalized as $\pi_k^{(i)} := 1/M \ (\forall i)$, where " $:=$ " denotes substitution. When resampling is not carried out, the following is executed:

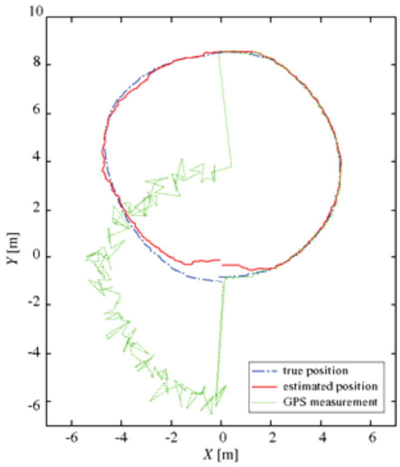
$$\boldsymbol{x}_k^{(i)} := \tilde{\boldsymbol{x}}_k^{(i)}, \ \pi_k^{(i)} := \tilde{\pi}_k^{(i)} \quad (27)$$

From these procedures, a new set of particles $\{\boldsymbol{x}_k^{(i)}, \pi_k^{(i)}\}_{i=1}^M$ is constructed. Then, the sampling is repreated with $k := k + 1$.

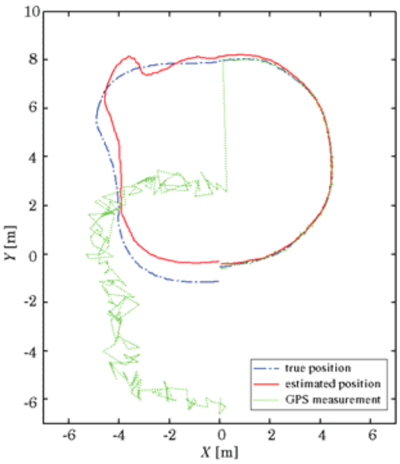
4 SIMULATION

4.1 Conditions

A robot on the field is controlled to pass eight targets in order. Here, when the robot's estimated position reaches the distance of 0.5 [m] from a target, the current target is switched to the next target. The velocity was set constant as $v_k = 10$ [m/s]. Furthermore, each parameter peculiar to the robot was defined as follows: length of wheel base is $l = 1$ [m], noise parameters of velocity $\alpha_1 = \alpha_3 = \alpha_5 = 3.0$,



(a) Proposed method



(b) Conventional method

Fig. 3 Trajectories of the estimation, measurement, and true state values on the third track using (a) proposed method and (b) conventional method.

noise parameters of steering $\alpha_2 = \alpha_4 = \alpha_6 = 0.3$, feedback gain $k_\phi = 0.8$, and the sampling period $\Delta = 0.01$ [s].

Each parameter of the noise in a GPS signal was defined as follows: $\sigma_b^2 = 0.0025$, $\sigma_p^2 = 0.01$. Furthermore, it was assumed that the area of $x < 0$, i.e., half of the field, was a shadow area and the bias and variance of a Gaussian noise added to the GPS signal in the area were given as follow: $r_k = 4$ [m] (const.), $\sigma_s^2 = 0.1$. The number of particles of the PF was $M = 1000$ and $ESS_{\text{th}} = 5.0$. Furthermore, for comparison with the proposed method, the simulation was performed using the following composition of a general PF, i.e., the sampling follows a system mode:

$$\tilde{\boldsymbol{x}}_k^{(i)} \sim p_q\left(\tilde{\boldsymbol{x}}_k^{(i)} \middle| \boldsymbol{x}_{k-1}^{(i)}\right) = p_f\left(\tilde{\boldsymbol{x}}_k^{(i)} \middle| \boldsymbol{x}_{k-1}^{(i)}\right). \quad (28)$$

Moreover, only the likelihood function is used when the direct detection of the GPS signal can be carried out, i.e.,

$$p_h\left(\boldsymbol{y}_k^{\text{GPS}} \middle| \boldsymbol{x}_k^{(i)}\right) = p_{\text{direct}}\left(\boldsymbol{y}_k^{\text{GPS}} \middle| \boldsymbol{x}_k^{(i)}\right). \quad (29)$$

The trajectories of the robot depended on the conventional methods and proposed methods one are shown in Fig. 3(a) and (b), respectively. These results indicate that the state estimation in the shadow area is accurate when the proposed method is used and a suitable control is executed. In contrast, owing to the accumulation of errors in the kinematics mode, the robot deviated significantly from the target trajectory when the conventional method was used. This is attributable to the error of state estimation being amplified by the feedback control.

5 CONCLUSION

We proposed a method to improve the state estimation accuracy of mobile robots near the high-rise buildings using the statistical property of reflected and diffracted waves of a GPS signal. A novel state observer using the KF and PF was constructed and installed into a feedback control system. Furthermore, the effectiveness of the control system for the autonomous mobile robot was shown via simulation.

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